Shielding of Gravitationally Induced Electric Fields*

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This paper proposes for the surface of a metal a model that is capable of resolving some of the experimental differences surrounding the measurement of gravitationally induced electric fields. Theoretical calculations of the magnitude of this field suggest that it should be of the order of $\mathbf{E}=-M\mathbf{g}/e$ where M is the mass of the lattice atom, $-e$ the electron charge, and $\mathbf{g}$ the acceleration due to gravity. The experiments of Beams and Craig agree with this theoretical prediction, but the measurements of Witteborn and Fairbank show a much smaller field $\mathbf{E}_1=+m\mathbf{g}/e$ where m is the electron mass. It is possible that these experimental differences are due in part to differences in surface conditions, and we show that a distribution of metallic whiskers on the metal surface can effectively shield the field $\mathbf{E}$ so that only the field $\mathbf{E}_1$ remains at distances from the metallic surface large compared to the whisker dimensions.

I. INTRODUCTION

The subject of gravitationally induced electric fields near metals is of current experimental and theoretical interest.[1–4] According to a theory of Schiff and Barnhill[5] there will be a gravitationally induced electric field $\mathbf{E}_1=-(mg/e)\hat{\mathbf{z}}$ near the surface of a metal standing in the gravitational field $-g\hat{\mathbf{z}}$, where m and $-e$ are the electron's mass and charge. Such a field would produce a force on an electron equal and opposite to its weight, and would account for the experimental results of Witteborn and Fairbank[1] on the free fall of the electron. It was pointed out by Dessler, Michel, Rorschach, and Trammell (DMRT)[6] however, that for the model considered by Schiff and Barnhill, proper account had not been taken of the effect of the compression of the metal by its weight, and when this effect was included, the gravitationally induced electric field in normal metals was shown to be of the order of $\mathbf{E}\sim+(Mg/e)\hat{\mathbf{z}}$, where M is the atomic mass. This result is of the opposite sign and some three or four orders of magnitude larger than that given in Ref. 5.

Schiff and Barnhill had computed the gravitationally induced electric field in an indirect manner making use of a reciprocity principle. Herring[7] showed that if proper account were taken of lattice compression, the reciprocity-principle approach of Schiff and Barnhill led to the same result as the direct approach of DMRT.

Beams[2] has determined the electric field at the surface of a rapidly spinning metallic rotor. The effective gravitational fields were several orders of magnitude larger than the earth's field, and the sign and magnitude of the induced electric field was in agreement with the estimates of the theory of DMRT.

Craig[3] has measured the variation of work function with stress in various metals, and this can be directly related to the gravitationally induced electric field. He too found agreement with the DMRT estimates as regards sign and order of magnitude of the field.

The question then arises as to how the large field predicted by all theoretical calculations and observed in the experiments of Beams and Craig can be cancelled in the apparatus of Witteborn and Fairbank leaving only the small field $\mathbf{E}_1$ which they found to act on the electrons in their experiment. Several authors[3,4] have suggested that there might be specific surface effects which shield the exterior of the metal from the large interior field $\mathbf{E}$ estimated by DMRT, leaving only the Schiff and Barnhill field $\mathbf{E}_1$ acting on the electrons in the Witteborn and Fairbank experiment.

II. "WHISKER" MODEL

Without commenting on previously advanced proposals, we wish to suggest a model which can account for an induced field of the order of the Schiff and Barnhill result outside a metal surface. Near (and outside) a vertical metal surface

$$\varphi(z_2)-\varphi(z_1)=-(1/e)\left(W(z_2)-W(z_1)\right)+(mg/e)(z_2-z_1), \quad (1)$$

where $\varphi(z)$ is the electrostatic potential at altitude z and $W(z)$ is the work function of the adjacent metal surface. To avoid the large field $\mathbf{E}$ estimated for the model of DMRT it must be assumed that W is independent of z. Now the field $\mathbf{E}$ comes from the effect of the differential compression of the metal under its own weight, assuming that the metal tube is supported mainly at one of its ends. The tube could be supported properly along its lateral surface so that the strain and therefore W would be independent of z. Furthermore, a thin foil could probably be made to adhere to a differentially stressed surface in such a way that it would not exhibit the differential strain of the underlying metal[3]; however we believe that another model given below provides a less-strained explanation for the phenomenon than that suggested by thin foils.

We adopt a model in which the vertical metal surface supports an array of long thin whiskers (See Fig. 1). Since there will be very little compression of the free ends of the whiskers, the work function will be independent of altitude at the ends of the whiskers. If the whiskers are sufficiently dense, the region beyond the whiskers will be well shielded from the strong average field $\mathbf{E}$ near the vertical metal surface, leaving only the field $\mathbf{E}_1$ of Schiff and Barnhill. In Sec. III, we will

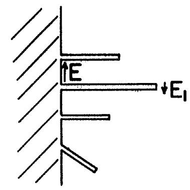

FIG. 1. Schematic representation of whiskers protruding from a vertical metal surface. The field at the surface is the DMRT field $\mathbf{E}\simeq M\mathbf{g}/e$. The field at the unstressed end of the whisker is the Schiff-Barnhill field $\mathbf{E}_1=-m\mathbf{g}/e$.

consider the conditions under which this "whisker model" can achieve this result.

III. STRAIN FIELD IN "WHISKER"

We will first show that if a whisker is attached to a stressed metal surface, then the strain in the whisker relaxes very rapidly to zero as a function of the distance along the whisker from the junction, divided by its width (St. Venant's principle).

In Fig. 2 we show an idealized model of a long thin whisker (beam of constant lateral dimension $2a$) attached at $x=0$ to the vertical surface of a metal slab (or a cylinder or prism) which is standing in a gravitational field. We suppose that the slab is supported at the top or bottom, and assume for definiteness that, except within a few whisker widths of the junction, only σ_{zz} differs from zero in the slab (σ_{ij} is the stress tensor), and that σ_{zz} is given by $\sigma_{zz}=\sigma_{zz}(0)-\rho gz$, where ρ is the density of the metal and g is the acceleration of gravity. The stresses and strains are continuous at the junction of the beam (whisker) and the slab so that in the neighborhood of the junction the beam shares the stress of the slab at the altitude z_0. We take a sufficiently small such that the relative change in σ_{zz} in an altitude change of the order of $\Delta z=a$ is negligible, i.e., $\rho ga\ll\sigma_{zz}(z_0)$. For the present we neglect the weight of the beam (whisker). Under these conditions the stress-strain distribution in and near the whisker is the same as in the situation pictured in Fig. 3 wherein a slab of height $2b(b=\text{few}a)$ is subject to uniform surface stresses, $F_z=\sigma_{zz}(z_0)\gg\rho gb$, on its horizontal faces at $z=\pm b$, the effect of the graviational body forces $-g\rho\hat{\mathbf{z}}$ in the slab and the beam being neglected. Neglecting the forces of gravity, the equation of equilibrium within the metal becomes

$$\partial_j\sigma_{ij}=0, \tag{2}$$

where $\partial_j=\partial/\partial x_j$, roman indices run from 1 to 3 labeling Cartesian coordinates, and the summation convention for repeated indices is understood.

We assume an isotropic stress-strain relation for simplicity

$$\sigma_{ij}=\lambda\xi_{ll}\delta_{ij}+2\mu\xi_{ij}. \tag{3}$$

The stress tensor ξ_{ij} is defined by

$$\xi_{ij}=\tfrac{1}{2}(\partial_j\xi_i+\partial_i\xi_j), \tag{4}$$

with $\xi_i(x,y,z)$ the vector displacement of the point which in the unstrained state is at $\mathbf{x}$.

Equations (2) and (3) together with the boundary conditions

$$\sigma_{ij}(s)n_j(s)=F_i(s), \tag{5}$$

where s denotes a surface point, $\hat{\mathbf{n}}(s)$ the unit surface normal vector, and $\mathbf{F}(s)$ the applied surface force per unit area, serve to determine the equilibrium state of the body. Alternatively, from (2)–(4) the Beltrami relations

$$(1+\sigma)\nabla^2\sigma_{ij}+\partial_i\partial_j\Theta=0 \tag{6}$$

follow [where σ is Poisson's ratio, $\sigma=\lambda/2(\lambda+\mu)$, and Θ is the trace of the stress tensor, $\Theta=\sigma_{ii}$], which together with (2) and (5) determine the equilibrium state in terms of the stresses alone.

We can obtain an estimate of the rapidity of decrease of stress with distance x along the beam by noting that the stress in the beam ($x>0$) is determined by the stresses on the surface S (denoted by the dotted lines in Fig. 3) obtained by projecting the beam back into the metal. Our problem then becomes that of finding the stress for $x>0$ in a long beam of constant cross section subject to certain surface forces $F(s)$ for $x<0$.

Let us suppose the beam (consisting of the actual beam for $x>0$ and its projection for $x<0$) is of a rectangular cross section with faces at $z=\pm a$, $y=\pm b$. Then if we suppose that the faces at $z=\pm a$ are subject to forces which are independent of y and that the faces at $y=\pm b$ are constrained not to move (except for motion in the plane), then the stresses and strains become independent of y and the solution of the resulting equations (2)–(6) in two independent variables is readily obtained.[8] A more realistic model for our purposes might be one in which the faces at $y=\pm b$ are free of normal stresses; but the solution of the equations for this case is considerably more involved than for the constrained case, and in any event it seems clear that we will underestimate the rate of decrease of stress with distance along the bar (for $x>0$) by imposing the constraint.

We suppose that there are forces $F_z(x,a)=-F_z(x,-a)$, $F_x(x,a)=F_x(x,-a)$, per unit area acting on the faces $z=\pm a$. It follows from (2)–(6) that the stress and strain tensors are biharmonic functions

$$\nabla^2\nabla^2\sigma_{ij}=\nabla^2\nabla^2\xi_{ij}=0, \tag{7}$$

which can be expanded in terms of the elementary solutions of (7),

$$\sigma_{ij}=(A_{ij}\cosh kz+A_{ij}'\sinh kz+B_{ij}z\sinh kz+B_{ij}'z\cosh kz)e^{ikx}. \tag{8}$$

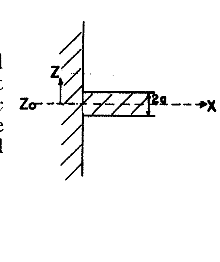

FIG. 2. Whisker of width $2a$ is located on the surface at the height z_0. It protrudes from the surface along the x axis and is subjected at its base to the gravitational stress at the bulk metal surface.

Substituting (8) into (2)–(6) we obtain

$$\sigma_{zz}=\int_{-\infty}^{\infty} dk(A_k \cosh kz+B_k z \sinh kz)e^{ikx},$$

$$\sigma_{xx}=-\int_{-\infty}^{\infty} dk[(A_k+2k^{-1}B_k)\cosh kz+B_k z\sinh kz]e^{ikx},$$

$$\Theta=-2(1+\sigma)\int_{-\infty}^{\infty} dk\, k^{-1}B_k \cosh kz e^{ikx}, \tag{9}$$

$$\sigma_{xz}=i\int_{-\infty}^{\infty} dk[(A_k+k^{-1}B_k)\sinh kz+B_k z\cosh kz]e^{ikx},$$

where

$$B_k=-k(ka+\tfrac{1}{2}\sinh 2ka)^{-1}(F_{zk}\sinh ka+iF_{xk}\cosh ka),$$

$$A_k=(ka+\tfrac{1}{2}\sinh 2ka)^{-1} \times[(\sinh ka+ka\cosh ka)F_{zk}+iak\sinh ka F_{xk}], \tag{10}$$

with

$$F_z(x,a)=\int dk\, F_{zk}e^{ikx},\qquad F_x(x,a)=\int dk\, F_{xk}e^{ikx}. \tag{11}$$

Since

$$F_z(x,a)=F_x(x,a)=0 \quad\text{for}\quad x>0,$$

F_{zk} and F_{xk} are analytic in the upper-half k plane, and the asymptotic behavior of the stress-strain tensors according to (9) and (10) will be determined by the roots of $ka=\frac{1}{2}\sinh 2ka$ with the smallest imaginary part. These roots are

$$2ka=\pm 2.2+i4.2, \tag{12}$$

and therefore for large x

$$\sigma_{ij}\sim c_{ij}(z)\cos[(2.2x/a)+\alpha]\exp[-4.2(x/2a)]. \tag{13}$$

Now the constraining forces on the Y faces of the beam are $\sigma_{yy}=\sigma(\sigma_{xx}+\sigma_{zz})$, as follows from (2) and the condition $\xi_{yy}=0$. Thus if the Poisson ratio σ is zero our solution also represents the case in which $\sigma_{yy}=0$. For nonzero σ it is reasonable, although we have not proved it, to assume that if the constraint on the y faces of the beam are relaxed then the stresses in the beam for $x>0$ will drop off even faster than that indicated by Eq. (13), and so the stress and strain in the whisker may be neglected at distances greater than a few whisker widths from the junction.

Returning now to Eq. (1), the contribution of the work function to the potential difference near $x=0$ is

$$-e^{-1}(W(z_2)-W(z_1)) \simeq -e^{-1}(\partial W/\partial\rho)[\delta\rho(z_2)-\delta\rho(z_1)]. \tag{14}$$

According to Ref. 6, this is $\sim(Mg/e)(z_2-z_1)$, some four orders of magnitude larger than the Schiff–Barnhill term in Eq. (1), $(mg/e)(z_2-z_1)$. But if points (z,x) near whiskers are considered, then the work function will contribute

$$-e^{-1}(W(z_2,x_2)-W(z_1,x_1)) \simeq -e^{-1}(\partial W/\partial\rho)[\delta\rho(z_2,x_2)-\delta\rho(z_1,x_1)] \tag{15}$$

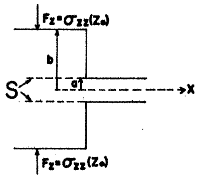

FIG. 3. Gravitational stress on the base of the whisker at z_0 has been replaced by an equivalent set of forces $F_z=\sigma_{zz}(z_0)$ (see text).

to $(\varphi(z_2,x_2)-\varphi(z_1,x_1))$, and according to Eq. (13), if $x_2/2a_2$, $x_1/2a_1>\sim 3$, (15) is reduced by about five orders of magnitude below (14), and for such points

$$\varphi(z_2,x_2)-\varphi(z_1,x_1)\simeq(mg/e)(z_2-z_1). \tag{16}$$

We have so far neglected the weight of the whisker, but the influence of its weight can be estimated. A whisker will be bent under its own weight, and this will lead to a compression of its lower surface relative to its upper surface. This gravity-induced strain and resulting work function change is, however, independent of the altitude of the whisker and will lead to no average gravitationally induced electric fields. Similarly the compression gradient along the length of the whiskers which are not horizontal is independent of the altitude of the whisker and thus does not contribute to the average vertical field.

IV. ELECTROSTATIC SHIELDING BY WHISKERS

We have shown that the strains, and therefore the work functions and electrostatic potentials [except for the $+(mg/e)z$ term], are independent of altitude for points near the whisker and a few whisker widths from its junction with the metal surface. The question now is, how dense an array of whiskers must we assume in order that the region beyond the whiskers will be sufficiently well shielded from the strong fields which are expected near the surface? Near the surface at a certain altitude let us take the potential as ϕ_0. We take as models of the whiskers long circular cylinders that are perpendicular to the plane metal surface, and to avoid inessential complication, we shall take $\varphi=0$ at the cylindrical surfaces right down to the junction. We take the radii of the cylinders to be a. Let the cylinders be disposed over the plane (which we take as the $x=0$ plane) in a regular planar array. Then Wigner-Seitz cells may be drawn around each cylinder, and the electrostatic problem becomes that of finding the solution of

$$\nabla^2\varphi=0 \qquad \text{for } x>0, \qquad r>a$$

in a given cell with the conditions

$$\varphi=\varphi_0 \quad\text{at}\quad x=0, \qquad r>a;$$

$$\varphi=0 \quad\text{at}\quad r=a;$$

and

$$\partial\varphi/\partial n=0$$

at the cell surface. For simplicity we consider a hexagonal array of whiskers. If the distance between nearest-neighbor cylinder axes is $2b$, then the cells are regular hexagonal prisms whose opposite faces are separated by $2b$. To simplify the problem we make the Wigner-Seitz approximation of replacing the hexagonal prisms by circular cylinders of radius $r=b$ (we should take r about 4% larger to make the cross-sectional areas the same).

The solution is of the form

$$\varphi=\sum_{n=0}^{\infty} A_n[Y_0(k_na)J_0(k_nr) - Y_0(k_nr)J_0(k_na)] \exp(-k_nx), \quad (17)$$

with the k_n's satisfying

$$Y_0(k_na)J_1(k_nb)-Y_1(k_nb)J_0(k_na)=0, \quad (18)$$

where J_ν and Y_ν are Bessel functions of the first and second kind, respectively. The coefficients A_n are determined in the usual way by the value of $\varphi(r, 0)$.

The value of φ for large x is determined by the $n=0$ term in (17). Equation (18) may be solved numerically for k_0b as a function of b/a and we obtain $k_0b=1.9$, 1.4, and 1.1, for $b/a=3$, 5, and 10, respectively. For $b/a\gg 10$

$$k_0b\simeq(\tfrac{1}{2}\ln b/a-\tfrac{1}{4})^{-1/2},$$

which gives $k_0b\approx 0.7$ for $b/a=100$. For large x we obtain

$$\varphi(x)\sim\phi_0 \exp(-2k_0bx/2b), \quad (19)$$

where φ_0 is the value of φ at $x=0$ $(r>a)$. If the whiskers are of length l we have

$$\varphi(l)/\varphi_0\approx \exp[-(2k_0b)(l/2b)]. \quad (20)$$

Taking as the shielding criterion $\varphi_l/\varphi_0<10^{-4}$, we obtain

$$l/2b>10/2k_0b, \quad (21)$$

or $l/2b>2.7$, 4.5 for $b/a=3$ and 10, respectively.

V. CONCLUSIONS AND DISCUSSION

The whisker model proposed here produces an effective conducting surface that is "supported from the sides" by shear forces and which greatly reduces the dependence of surface strain on altitude. In terms of the concept of the "surface pinch" introduced by Herring,[7] this model provides a shielding by preventing the field of a test charge from acting on the lattice ions. Charges can flow along the whiskers to reduce greatly the "surface pinch."

Does this model have any applicability to the real surfaces involved in the experiments of Witteborn and Fairbank? It should first be noted that various modifications of the model used for these calculations are possible. The results would be unchanged, for example, if the whiskers were tangled. Similarly, if rather than whiskers, the surface more nearly resembled slabs of width a, spacing b, and length l, then so long as $l/a\gtrsim 3$, the differential strain at the end of the slabs would be effectively zero, and if $l/b\gtrsim 3$, then the slabs would provide the necessary shielding.

The 3-5 length-to-spacing ratio of the whiskers or slabs which would be required to shield the DMRT field E by the required factor of about 10^{-4} would correspond to a moderately heavy "beard" of whiskers on the face of the metal. It should be possible to detect whiskers of this size and density by appropriate electron-microscope techniques. Such whiskers are known to grow on the surface of a large class of metals under appropriate conditions of strain and temperature.[9] It seems unlikely that these conditions would have been met for the copper tubes used by Witteborn and Fairbank.[1] Nevertheless, it is possible that the electroforming process used to produce the tubes could have produced a surface with the required properties. Since the drift tubes were "electroformed onto a polished aluminum mandrel which was later dissolved away,"[1] it seems conceivable that whiskers might have grown during the etching process. This would also explain the absence of the shielding in the experiments of Beams and of Craig.

In the experiments of Witteborn and Fairbank, the electric field acting on the electron could be varied by passing a current through the copper tube. The whisker model permits the existence of such fields outside the drift tube, since the ir potential drop within the metal will appear unshielded at the surface of the whiskers, while the strain-induced fields will be shielded as previously discussed.

ACKNOWLEDGMENTS

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